

# Performance Analysis of a Cost Effective Modified FourTree Network

Sonia Sharma\*, Gurjeet Walia\*\*

\*sonia19832@yahoo.co.in, \*\*walia.pbi.uni@gmail.com

*Abstract-In this paper we are going to study a new network named as Modified FourTree-New(MFT-New) Network and will compare its performance with ASEN-2(Augmented Shuffle Exchange Network) and FT(FourTree Network) and Modified FourTree. MFT-New Network is multipath, dynamic MIN which has been derived from Modified FourTree (FT) Network. MFT-New has one extra stage and eight switches more than MFT Network. In spite of the several available performance metrics, we will mainly emphasize on Probability of Acceptance, Bandwidth, Cost and Bandwidth per unit cost.*

## I. INTRODUCTION

Interconnection networks used for processor-memory communication are often required to perform simultaneous connections for many input-output pairs. Many irregular MINs have been introduced and studied in terms of reliability, probability and permutation passibility. In this Paper, an irregular, multipath network named “Modified FourTree-New Network” has been studied and its probability analysis has been carried out. Also, a comparison has been performed among different MINs to find out which is better in context of which parameter. For the Probability Analysis, probability equations for MFT have been derived. The parameters involved here are Bandwidth, Probability of Acceptance, Cost and Bandwidth per unit cost. The expressions for probability equations of all the stages of the MFT have been derived and the parameters Bandwidth, Probability of Acceptance, Cost and Bandwidth per unit cost are computed on the basis of these equations. Bandwidth of MFT is compared with the bandwidth of some of the regular and irregular MINs. Bandwidth per unit cost is derived by simply dividing the bandwidth of MIN by the cost factor of the corresponding MIN.

## II. EXTRA STAGE CUBE

The ESC[3] is formed from the Generalized Cube by adding an extra stage along with a number of multiplexers and demultiplexers. The extra stage, stage  $n+1$ , is placed on the input side of the network and implements the cube<sub>0</sub> interconnection function. Stage  $n+1$  and stage 1 can each be enabled or disabled (bypassed). A stage is enabled when its interchange boxes are being used to provide interconnection. It is disabled when its interchange boxes are being bypassed. Enabling and disabling in stages  $n+1$  and 1 is accomplished with a demultiplexer at each box input and a multiplexer at each output. The demultiplexer and multiplexer are configured such that they either both connect to their box(enable) or both

shunt it(disable). Stage enabling and disabling is performed by a system control unit. The ESC MIN, for  $N = 8$  is shown in Figure 1.

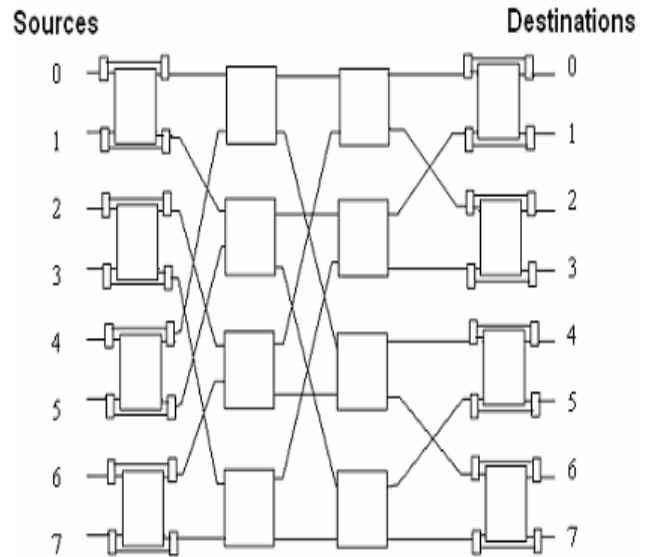


Figure 1: An  $8 \times 8$  ESC Network

## III. AUGMENTED SHUFFLE EXCHANGE NETWORK(ASEN)

ASEN[4] is a regular MIN, has the same number of switching elements(SEs) in each stage. A  $16 \times 16$  ASEN has  $(\log_2 N - 1)$  stages where the initial  $(\log_2 N - 2)$  comprise of  $N/2$  SEs of size  $3 \times 3$  and the terminal stage having  $N/2$  SEs of size  $2 \times 2$  switches. Each source has a primary multiplexer of size  $2 \times 1$  with a SE. Equal number of  $1 \times 2$  demultiplexers are provided before the destinations. For every loop of switches, another loop exists, connected to the same set of switches in the next stage. For this self routing network, each source has a primary multiplexer and a SE, along with a secondary multiplexer with a SE. Each source attempts entry into the ASEN through its primary multiplexer and SE. If either of them has a fault, the request is sent to the other one. In case secondary multiplexer also has a fault, then failure occurs in the MIN and request drops out. Similar is the case for the demultiplexers. There are several versions of the ASEN, depending upon the number of switches included in each loop. Here, the discussion has been limited to ASEN-2, in which the loops contain exactly two switches. A  $16 \times 16$  ASEN-2 is shown in Figure 2.

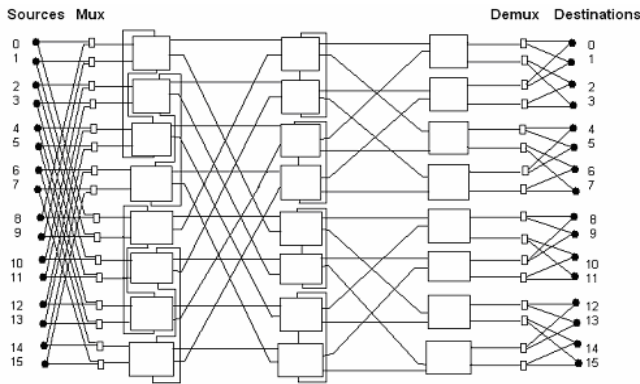


Figure 2: A 16 × 16 ASEN-2 Network

#### IV. FORTREE NETWORK

The FT[1] Network of size  $2^n \times 2^n$  contains two identical groups  $G^0$  and  $G^1$ . The two groups are formed on the most significant bit (MSB) of the source destination terminals. Thus, half of the source destination terminals with MSB 0 will fall into group  $G^0$  and others having MSB 1 will fall into group  $G^1$ . The  $i^{\text{th}}$  and  $(2m-i)^{\text{th}}$  stage will have exactly  $2^{n-1}$  switches. The switches in a stage having same position are chained together to form a loop, except last stage. Each source and destination is connected with multiplexers and demultiplexers. Thus, a FT network of size  $2^n \times 2^n$  consists of  $(2m-1)$  stages and total  $(2^{m+2}-6)$  switches with  $2^{n-1}$  of size  $2 \times 2$  and rest of size  $3 \times 3$ . There are  $2^n$  multiplexers and equal number of demultiplexers of  $2 \times 1$  and  $1 \times 2$  respectively. A  $16 \times 16$  FT Network is shown in Figure 3.

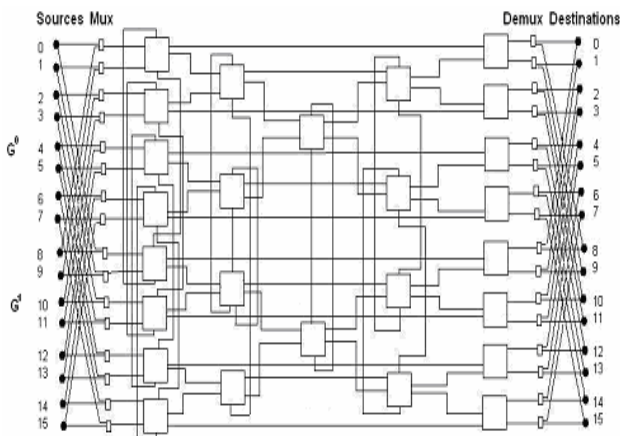


Figure 3: A 16 × 16 FourTree Network

#### V. MODIFIED FORTREE NETWORK

Modified FourTree Network[1] is a network derived from FT network with one stage less than FT Network. Its construction resembles to FT and linkage resembles to ASEN. It contains multiplexers and demultiplexers same as that of

FT. It consists of total  $(2^{m+2}-8)$  switches with  $2^{n-1}$  of size  $2 \times 2$  and rest of size  $3 \times 3$ . The  $2 \times 1$  multiplexers are connected in such a way that each multiplexer between pair of sources have all the bits  $(s_{n-1} \dots s_2 s_0)$  equal except the bit  $s_1$ . The  $2 \times 1$  demultiplexers are so connected that each demultiplexer is connected to destination having all the bits  $(d_{n-1} \dots d_1)$  same. The demultiplexers are connected as in case of ASEN-2. A  $16 \times 16$  Modified FourTree Network is shown in Figure 4.

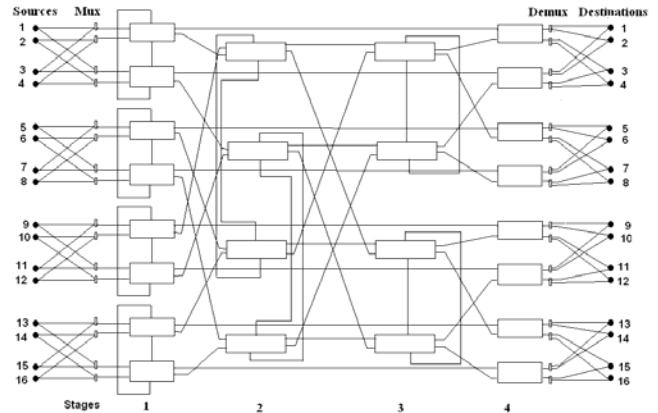


Figure 4: A 16 × 16 FourTree Network

#### VI. MFT-NEW NETWORK

MFT-New is a network derived from MFT network with one extra stage MFT Network. It contains multiplexers and demultiplexers same as that of FT. It consists of total  $(2^{m+2}-8)$  switches with all of size  $3 \times 3$ . The  $2 \times 1$  multiplexers are connected in such a way that each multiplexer between pair of sources have all the bits  $(s_{n-1} \dots s_2 s_0)$  equal except the bit  $s_1$ . A  $16 \times 16$  MFT-New Network is shown in Figure 5. It can be clearly observed from the Figure 5 that this network have the intermediate stage as in case of FT network, but the difference is that this intermediate stage has number of switches equal to first and last stage as in the case of MFT network. Since the number of switches are greater than the preceding and succeeding stages, so this stage will not be a bottleneck. Rather it will act like a bridge between the stages. The benefit of including this stage is that it improves the bandwidth and finally the performance of MIN. This makes MFTNew better from MFT.

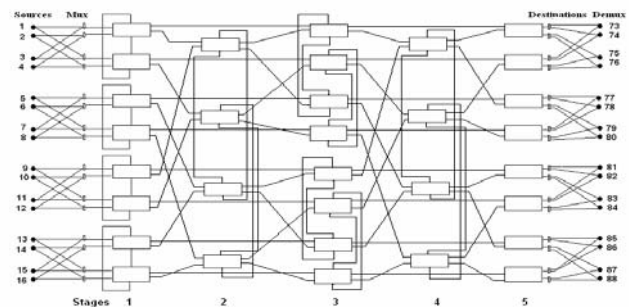


Figure 5: A 16 × 16 MFT-New MIN

## VII. BASIS OF DERIVATION

For the derivation of the probability equations, consider a MIN of size  $a^n \times b^n$  with  $a^n$  sources and  $b^n$  destinations. The analysis is based on  $a \times b$  crossbar switches. It is assumed that all the destinations are independently and identically distributed. If one switch is used by any processor or module, it can't be accessed by any other processor. The probabilistic approach is based on "Uniform Reference Model".

Let the request generation probability, at each of the inputs of  $a \times b$  crossbar switches, is  $P$ . The expected number of requests that passes per unit of time is given by  $b-b(1-P/b)^a$ [2]. Dividing it by number of output lines, the rate of request at any of the output lines is obtained. So the output rate, which is function of input line is given by  $1-(1-P/b)^a$ . Since the output rate at one stage is input rate to next stage, the output rate can be recursively evaluated, starting from the initial stage. Output rate of the final stage will determine the Bandwidth of a MIN. So, Bandwidth  $BW$  is given by:-

$$BW = b^n p[n]$$

And  $p[0]=P$ .

where  $p[n]$  represents the probability for the last stage. Since the network under consideration, i.e. MFT-New, is an irregular MIN, the probability equations for different stages would not be the same. The probability, that the request made by a processor would reach the destination or not, depends on the structure of MIN. Probability Equations for MFT-New Network are given below.

$$\begin{aligned} p[1] &= 1 - (1 - p[0]/3)^3 \\ p[2] &= 1 - (1 - p[1]/6)^3 \\ p[3] &= 1 - ((1 - 2 * p[2]/3)^2 * (1 - p[1]/3)) \\ p[4] &= 1 - (1 - p[3]/6)^3 \\ p[5] &= 1 - ((1 - 2 * p[4]/3)^2 * (1 - p[3]/3)) \\ &\quad \& p[0]=P. \end{aligned}$$

To have an idea, how these equations have been derived, let us have a closer look at Figure 5. It has five stages. The first and the last and third stages have  $N/2$  switches whereas the second and fourth stages have  $N/4$  switches. As discussed above, the probability to reach  $i^{\text{th}}$  stage is given by  $1 - (1 - p_{i-1}/b)^a$ , where  $p_{i-1}$  is the probability that the request would reach the  $(i-1)^{\text{th}}$  stage. In the first stage, there are  $3 \times 3$  switches i.e.  $a=b=3$ . So, substituting values of  $a$  and  $b$ , the expression for  $p[1]$  is obtained. For the second stage, as the number of switches are the half of the number of switches in the first stage, the probability to reach the second stage would be the half, due to this reason,  $p[1]=p[1]/2$  is used and hence the equation.

Let us discuss the third stage now. From the figure, it can be observed that for the third stage, the input lines are coming from the first and second stages. The number of switches in the second stage is half the number of switches in the third

stage, so the probability to reach the third stage from the second stage would be the double and in the case of request coming from the first stage, the probability will have no changes due to the same number of switches. For the fourth stage, the number of the switches is half of the previous one, that is why,  $p[3]=p[3]/2$  is used. In case of fifth stage, the input lines are coming from the third and fourth stages. The number of switches in the fourth stage is half the number of switches in the fifth stage, so the probability to reach the fifth stage from the fourth stage would be the double and in the case of request coming from the third stage, the probability will have no changes due to the same number of switches.

Now, the probability equations for MFT-New have been derived. The probability at all the stages can be easily calculated and hence several other performance parameters can also be calculated.

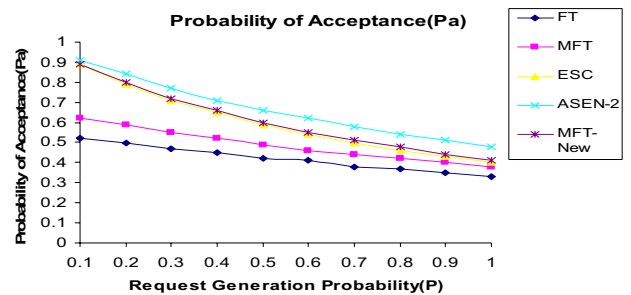
The Performance Parameters Bandwidth, Probability of Acceptance, Cost and Bandwidth per unit cost will be calculated using these equations and comparison with ESC, ASEN-2, FT and MFT will also be done.

### A. Comparison of Probability of Acceptance ( $P_a$ )

Here, the Probability of Acceptance of two regular and three irregular MINs has been calculated. Table and Graph show the Probability of Acceptance calculated for different values of request generation probability (where  $P$  is the request generation probability).

**Table 1: Probability of Acceptance vs. P**

P	FT	MFT	ESC	ASEN-2	MFT-New
0.1	0.52	0.62	0.89	0.91	0.89
0.2	0.5	0.59	0.79	0.84	0.8
0.3	0.47	0.55	0.71	0.77	0.72
0.4	0.45	0.52	0.65	0.71	0.66
0.5	0.42	0.49	0.59	0.66	0.6
0.6	0.41	0.46	0.54	0.62	0.55
0.7	0.38	0.44	0.5	0.58	0.51
0.8	0.37	0.42	0.46	0.54	0.48
0.9	0.35	0.4	0.43	0.51	0.44
1	0.33	0.378	0.4	0.48	0.41



**Graph 1: Comparison of Probability of Acceptance ( $P_a$ )**

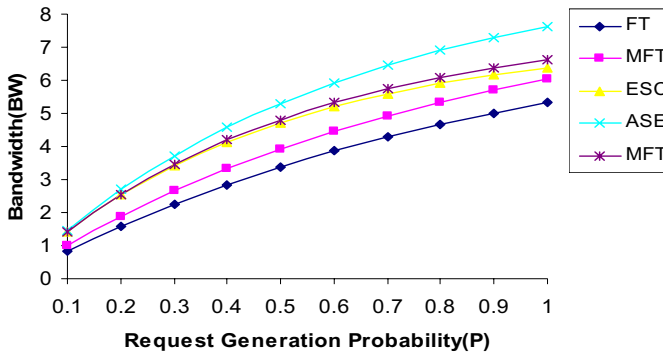
**B. Comparison of Bandwidth**

Now, the calculations for the Bandwidth of the five MINs have been carried out. Table and Graph represent the comparison of the Bandwidth of the five networks.

**Table 2: Bandwidth vs. P**

P	FT	MFT	ESC	ASEN-2	MFT-New
0.1	0.84	0.99	1.42	1.46	1.42
0.2	1.59	1.88	2.54	2.69	2.55
0.3	2.26	2.65	3.43	3.71	3.46
0.4	2.85	3.33	4.14	4.58	4.2
0.5	3.39	3.93	4.72	5.31	4.81
0.6	3.87	4.463	5.19	5.92	5.32
0.7	4.3	4.93	5.58	6.45	5.74
0.8	4.68	5.35	5.9	6.9	6.09
0.9	5.02	5.72	6.17	7.28	6.39
1	5.33	6.05	6.39	7.61	6.64

**Bandwidth Analysis**



**Graph 2: Comparison of Bandwidth**

**C. Comparison of Cost**

Here, the comparison will be done on the basis of the cost that MINs incur to interconnect the processors. Lower the cost, better the network will be. The expression for the cost parameter of the MINs is given in the table below.

**Table 4.3: Cost Parameters**

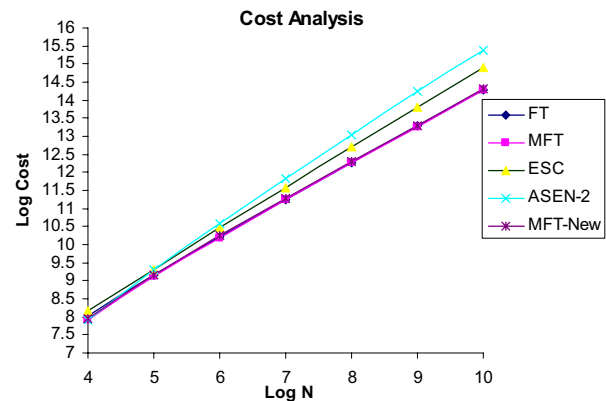
MIN	Cost
ESC	$2N(\text{Log}_2N+5)$
ASEN-2	$3N(1.5 \text{Log}_2N-1)$
FT	$(9.75 \cdot 2^{n+1} - 54)$
MFT	$(9.75 \cdot 2^{n+1} - 72)$
MFT-New	$(9.75 \cdot 2^{n+1} - 72) + N/2$

(Adopted from [5])

Let us have a comparative look at cost of the five MINs in the form of table and graph.

**Table 4: Cost vs. Log N**

Log N	FT	MFT	ESC	ASEN-2	MFT-New
4	8.011	7.91	8.17	7.91	7.95
5	9.15	9.11	9.32	9.29	9.15
6	10.22	10.2	10.46	10.58	10.24
7	11.25	11.24	11.58	11.83	11.28
8	12.27	12.26	12.7	13.04	12.3
9	13.28	13.27	13.8	14.23	13.31
10	14.28	14.28	14.91	15.39	14.32



**Graph 3: Cost vs. Log N**

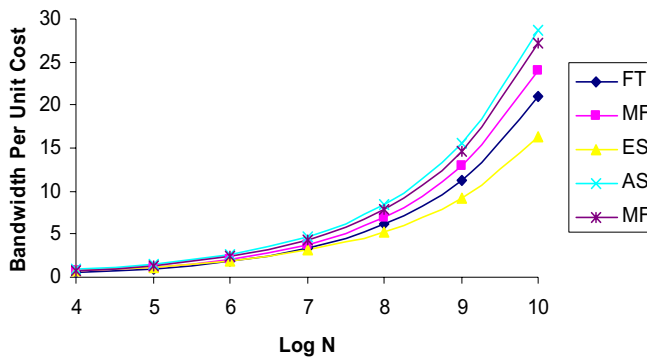
**D. Comparison of Bandwidth per unit cost**

Bandwidth per unit cost is obtained by dividing the bandwidth by the cost of the MIN. To make the comparison even, the request generation probability is fixed at P=0.8 and the parameter has been computed by varying the values of N i.e. number of processors. Higher is this parameter, better will be the MIN. Table and graph given below represent the Bandwidth per unit cost for the five MINs for various number of processors.

**Table 5: Bandwidth per unit cost vs. Log N**

Log N	FT	MFT	ESC	ASEN-2	MFT-New
4	0.58	0.68	0.72	0.87	0.77
5	1.02	1.17	1.15	1.49	1.33
6	1.83	2.1	1.88	2.61	2.38
7	3.33	3.81	3.13	4.66	4.32
8	6.1	6.98	5.31	8.46	7.92
9	11.2	12.9	9.12	15.51	14.64
10	20.9	23.9	16.4	28.68	27.22

### Bandwidth Per Unit Cost Analysis



**Graph 4: Bandwidth per unit cost vs. Log N**

#### VIII. Discussion

Graph 1 is representing Probability of Acceptance for ESC, FT, MFT and ASEN-2 MINs. From the Graph, it can be seen that Probability of Acceptance decreases as probability of request generation(P) increases. Among the discussed irregular Networks, MFT has higher probability of acceptance as compared to FT network. In case of regular MINs, ASEN-2 has higher probability of acceptance as compared to ESC as well as, it is better than rest of the three MINs. But if the fall in the curve is considered, then it is concluded that the fall in probability of acceptance is rapid for regular networks while it is less rapid for irregular networks. It means that performance of irregular MINs is more efficient than the regular MINs, even if the congestion increases very rapidly.

Graph 2 represents Bandwidth Analysis of the five MINs. From the Graph, it can be seen that among the discussed irregular Networks, MFT has higher Bandwidth gain as compared to FT network. In case of regular MINs, ASEN-2 has better Bandwidth gain as compared to ESC as well as, it is better than rest of the three MINs.

Graph 3 is representing Cost Analysis. Form the Graph, it can be concluded that Cost for ASEN-2 is higher as compared to FT, MFT and ESC. MFT network has Cost lower than FT network for small values of N, but if number of processors becomes high, both will behave alike.

Graph 4 is representing Bandwidth per unit cost. Form the Graph, it can be concluded that Bandwidth per unit cost is higher for ASEN-2 in case of regular MINs and is higher for MFT Network in case of irregular MINs. Also, MFT is better than the ESC in context of Bandwidth per unit cost.

#### CONCLUSION

It has been concluded that in case of Probability of Acceptance, Bandwidth and Bandwidth per unit cost, ASEN-2 is better than the rest of the three MINs, but it has the highest cost among all the five MINs. It has also been concluded that MFT network is better than FT network, from which it has been derived, in context of all the four parameters.

#### REFERENCES

- [1] Aggarwal Himanshu, and Bansal P.K., Routing and Path Length Algorithm for cost effective Modified Four Tree Network, I.E.E.E. Transactions on Computers, 2002, pp[293-294].
- [2] Bhuyan N. Laxmi, Yang Qing, and Agarwal P. Dharma, Performance of Multistage Interconnection Networks, I.E.E.E transactions on Computers, 1989, pp[395-397].
- [3] Adams III. G.B., Agarwal D.P., and Siegel H.J., The extra-stage cube: a fault tolerant interconnection network for super systems, I.E.E.E Transactions on Computers, 1982.
- [4] Kumar V.P., and Reddy S.M., Augmented Shuffle exchange multistage interconnection networks, Computers 1987.
- [5] Bansal P.K., Singh K., Joshi R.C, Routing and path length algorithm for a cost effective four-tree multi-stage interconnection network, International conference. I.E.E.E INFOCOM, 1991.