

# RELIABILITY ANALYSIS OF MULTISTAGE OPTICAL INTERCONNECT NETWORKS FOR HIGH SPEED MULTIPROCESSOR SYSTEM

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**Abstract:** Computer systems capable of teraflop computation are becoming a near-term reality with the emergence of high-speed powerful processors that can run cooperatively in a multiprocessor system.. The underlying fabric that supports communication among processors in the system is the interconnection network. The basic function of an interconnection network is to transfer information from the input nodes of the network to the output nodes by setting up communication paths for routing through the network, in an efficient manner. In this paper we study the reliability analysis of multistage interconnection network using optical interconnect following shuffle exchange network topologies in multiprocessor system Further the TR,BR,NR reliability is computed for 8X8 SEN,SEN+ and SEN+2.and result so obtained is used for computing  $MTTF*\lambda$ . Further NR is evaluated for larger size of network varying in size from 8X8 to 1024X1024.

**Keywords:** SEN ,SEN+,SEN+2, Reliability ,TR , BR , NR , MTTF.

**Introduction:** Interconnection networks play a major role in the performance of modern parallel computers. These networks can provide the communication in a parallel processing system consisting of a large number of processors that are working together to perform a single over all task. Multistage interconnection networks (MINs) are designed to provide fast and efficient

communication at a reasonable cost .Many different interconnection networks between the extremes of the single bus and the completely connected scheme have been proposed in the literature In general, MINs consist of layers of switching elements (SEs) with a specific topological pattern. These networks provide interconnection between the set of processors (inputs) and the set of memory modules (outputs). They fall within the category of indirect network as they rely on intermediate elements to provide the interconnection between the input and output elements. It has been extensively used in both circuit switching and packet switching networks with the introduction of buffered switches. The number of stages, interconnection topology, and the type of SEs used in the network configuration differentiate each MIN fault tolerant. Shuffle exchange network topology is followed for interconnection between processors and memory modules in multiprocessor system.

## **Network Description:**

Shuffle exchange network is just one network in large class of topologically equivalent MIN's that include omega, indirect binary n cube Figure 3 is an example of shuffle exchange network .Each switching element (SE) the basic building block of SEN , can be

viewed as 2X2 SEN .The SE can transmit inputs either straight through itself or exchange the inputs .The specific SEN's will examine will have  $N= 2^n$  inputs, termed sources and  $2^n$  outputs termed destinations .There is a unique path between each source destination pair. The SEN has n stages and each stage has  $N/2$  switching element. The network complexity , defined as the total number of switching elements in MIN is  $(N/2) \log(N)$  which for 8X8 SEN is 12 SE's. We will describe the position of switching element i in the stage j as  $SE_{ij}$  The SE is a self routing network. That is a message from any source to given destination is routed through the network according to the binary representation of the destination's address. Figure 1 shows the connection between S=000 to D=101

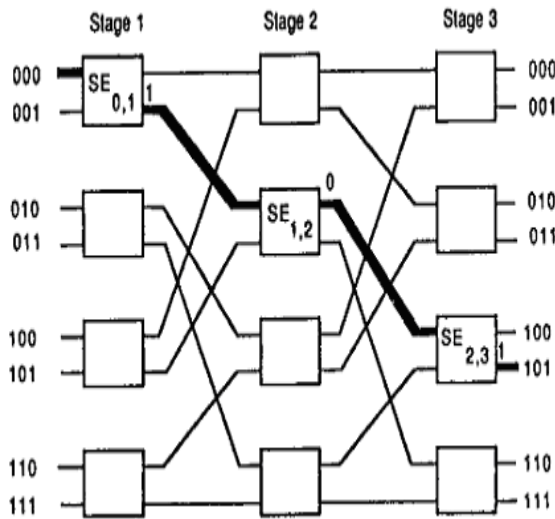


Fig 1 8X8 SEN showing routing between 000 to 101

An  $N \times N$  SEN+ network is an  $N \times N$  SEN with additional stage . Fig 2 shows 8X8 SEN+ stage The first stage (labeled stage 0) is additional

stage and will require implementation of different control strategy. The basic idea of adding a stage to the SEN is to allow two paths for communication between each source and destination .While the path in the first and last stages of the SEN+ are not disjoint, the path in the intermediate stages do transverse disjoint link..The network complexity is  $N/2(\log_2 N + 1)$

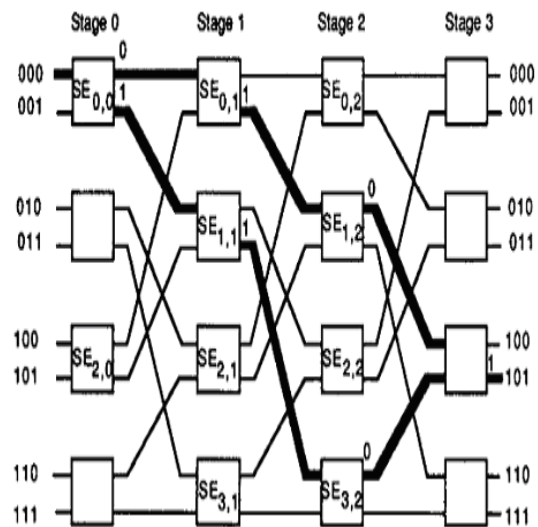


Fig2 SEN+ means shuffle exchange network with additional stage

In general, a SEN+2 consists of N inputs and N outputs,  $N/2$  SEs per stage,  $(\log_2 N)+2$  stages, and  $(N) ((\log_2 N)+3)$  links. The network complexity is defined as the total number of SEs in the MIN, that is,  $(N/2) ((\log_2 N)+2)$  which is 20 SEs for an 8X8 SEN+2. Fig 3. shows SEN+2 topology. From the reliability point of view terminal , broadcast and network reliability is computed and compared . In general, a SEN+2 consists of N inputs and N outputs,  $N/2$  SEs per stage,  $(\log_2 N)+2$  stages, and  $(N) ((\log_2 N)+3)$  links. The network complexity is defined as the total

number of SEs in the MIN, that is,  $(N/2) ((\log_2 N)+2)$  which is 20 SEs for an 8X8 SEN+2. The number of terminal paths between an input and an output switches will be increased to  $2k$  by adding  $k$  extra stages to the SEN.

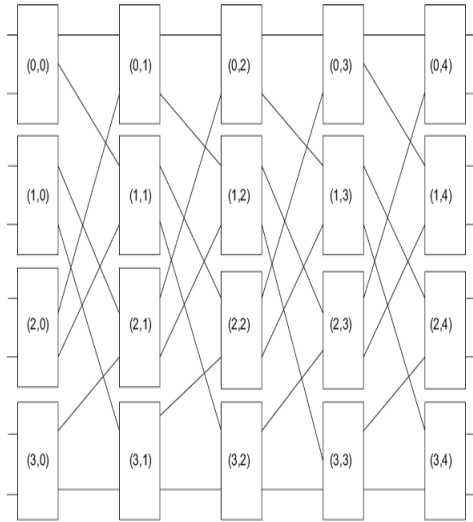


Fig3. SEN+2 means shuffle exchange network with two additional stages

**Reliability** The Reliability,  $R(t)$ , is defined as the probability that the component or system experiences no failures during the time interval zero to  $t_1$  given that the component or system was repaired to a like new condition or was functioning at  $t_0$ . The Unreliability,  $F(t)$ , of a component or system is defined as the probability that the component or system experiences the first failure or has failed one or more times during the time interval zero to time  $t$ , given that it was operating or repaired to a like new condition at time zero.

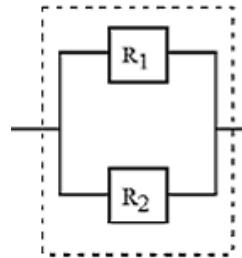
$$R(t) + F(t) = 1$$

Unreliability  $F(t) = 1 - R(t)$

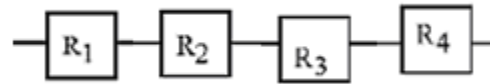
A Reliability Block Diagram (RBD) performs the system reliability and availability analyses on large and

complex systems using block diagrams to show network relationships. The structure of the reliability block diagram defines the logical interaction of failures within a system that are required to sustain system operation. The rational course of a RBD stems from an input node located at the left side of the diagram. The input node flows to arrangements of series or parallel blocks that conclude to the output node at the right side of the diagram. A diagram should only contain one input and one output node. The RBD system is connected by a parallel or series configuration. A parallel connection is used to show redundancy and is joined by multiple links or paths from the Start Node to the End Node.

$$R_p = (1 - (1 - R_1) (1 - R_2))$$



A series connection is joined by one continuous link from the Start Node to the End Node.  $R_s = R_1 R_2 R_3 = \prod R_i$



A system can contain a series, parallel, or combination of series and parallel connections to make up the network. Partial Active Redundancy system ( $k$  out of  $N$ ): In this all  $N$  units are available and simultaneously active; For the system to succeed, at least  $k$  out of  $N$  units must succeed.

–  $k = 1$ : simple parallel system

–  $k = N$ : series system

**Terminal reliability** is defined as the probability of successful communication between an input and

an output switches. In this section, terminal reliability of SEN, SEN+, compared. The SEN is a unique-path MIN that has N input switches and N output switches and n stages, where  $n = \log_2 N$ . Each stage consists of N/2 interchange boxes, where each box being controlled individually through routing tags. An eight-input/eight-output SEN with three stages, 12 SEs and 32 links is shown in Fig. 1 Let r be the probability of a switch being operational. Since SEN is a unique-path MIN, the failure of any switch will cause system failure, so from the reliability point of view, there are  $\log_2 N$  SEs in series for each terminal path. Hence, the terminal reliability of an NxN SEN is given by

$$R_t(\text{SEN}) = (r)^{\log_2 N}.$$

It is noted that there is only a single path between a particular input  $S_i = 1, 2, 3, 4$ , and a particular output  $D_i$  in the 8X8 SEN so the terminal reliability for  $N = 8$  can be calculated as

$$R_t(\text{SEN}) = (r)^2$$

SEN+ is a two-path MIN derived from the SEN by adding an extra-stage. Fig.2 shows an eight-input/eight output SEN+ with four stages consisting of 16 SEs and 40 links. Since the SEN+ is a two-path MIN, there are two connection paths between a particular input and output. From the reliability point of view, this system can be represented as a parallel system path, consisting of  $(\log_2 N) - 1$  SEs each. Where, each path is connecting the input and output SE in series. Hence, the terminal reliability of an NXN SEN+ is given by

$$R_t(\text{SEN}+) = (r)^2(1 - (1 - r^{(\log_2 N) - 1})^2).$$

By adding an extra-stage to the 8X8 SEN, the number of connecting paths between any input and output switches will increase to two. Therefore, the terminal reliability of the 8x8 SEN+ is higher than that of the 8X8 SEN.

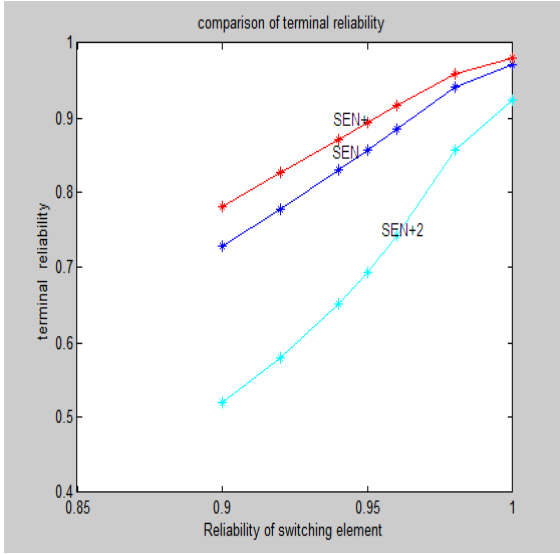
An 8X8 SEN+2 consists of eight inputs and eight outputs, four SEs per stage, five stages, and 48 links as demonstrated in Fig. 3. It is observed that there are four terminal paths between any pair of input  $S_i$  ( $i = 1, 2, 3$ , and 4) and output  $D_i$ . Suppose that the position of a SEs  $i$  in stage  $j$  is represented by  $SE_{i,j}$ . Since there are 20 SEs in the 8X8 SEN+2 and five stages (0, 1, 2, 3, and 4), the SEs are numbered from  $SE_{00}, SE_{10}, SE_{24}, SE_{34}$ . As an example, the terminal reliability between  $SE_{00}$ , and  $SE_{04}$  in Fig. 3 are examined

The terminal reliability of the 8X8 SEN with two additional stages for  $N = 8$  can be computed as follows:

$$R_t(\text{SEN} + 2) = r^{10} + 2r^9(1 - r) + 8r^8(1 - r)^2 + 8r^7(1 - r)^3 + 2r^7(1 - r)^2 + 4r^6(1 - r)^3 + 4r^6(1 - r)^2 + 4r^5(1 - r)^2.$$

Switching reliability	Terminal reliability of the SEN	Terminal reliability of the SEN+	Terminal reliability of the SEN+2
0.99	0.970299	0.979712	0.924345
0.98	0.941192	0.958894	0.856787
0.96	0.884736	0.915935	0.742687
0.95	0.857375	0.893921	0.694677
0.94	0.830584	0.871628	0.651818
0.92	0.778688	0.826431	0.579273
0.90	0.729000	0.780759	0.520995

Table 1 Terminal reliability comparison



Graph for terminal reliability comparison for SEN,SEN+,SEN+2

**Broadcast reliability** is defined as the probability that a single-input switch is able to broadcast data or connected to all the output switches. As an illustration, the broadcast reliability of SEN, SEN+for the 8X8 case are examined. Since SEN is a unique-path MIN that has N inputs and N outputs, there is only one broadcast path in this network. The broadcast reliability of the SEN can be calculated by assuming that the SE in the input stage and the SEs in the output stage have to work in order the network to be operational. Then by conditioning on the first stage and listing all possible combinations of paths between an input and all outputs, the broadcast reliability can be computed. The failure of any switch in a broadcast path will cause the system failure, so from the reliability point of view; all SEs in a broadcast path are critical and can be assumed as in series. Since a broadcast path in the 8X8 SEN consists of seven SEs, the broadcast reliability as a function of

the reliability of a SE for N =8 can be calculated as follows:

$$R_b(\text{SEN}) = (r)^7.$$

Since the SEN+ is a two-path MIN, there are two broadcast paths from every input to all outputs. The broadcast reliability of the 8X8 SEN+ for N =8 can be calculated by

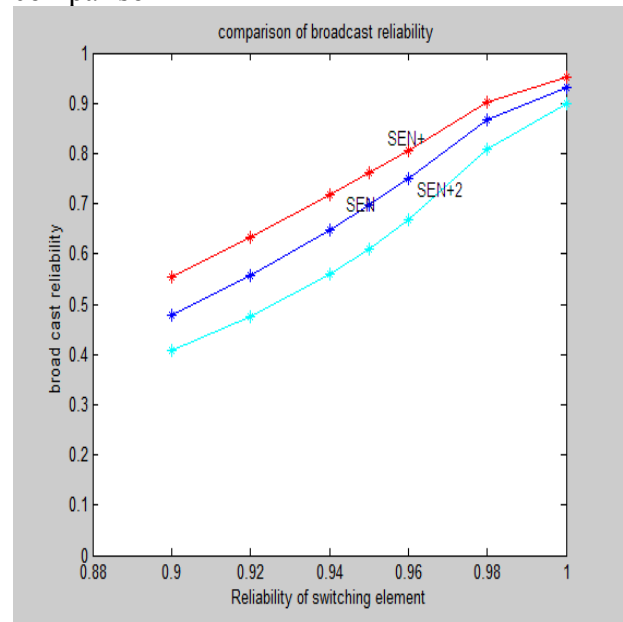
$$R_b(\text{SEN+}) = 2(r)^8 + 2(r)^9 - 4(r)^{10} + (r)^{11}.$$

The broadcast reliability of the 8X8 SEN with two additional stages for N = 8 can be calculated as follows:

$$R_b(\text{SEN} + 2) = r^{15} + 4r^{14}(1 - r) + 20r^{13}(1 - r)^2 + 32r^{12}(1 - r)^3 + 16r^{11}(1 - r)^4 + 10r^{11}(1 - r)^2 + 12r^{10}(1 - r)^3 + 4r^9(1 - r)^2.$$

SE reliability	Broadcast reliability of the SEN	Broadcast reliability of the SEN+	Broadcast reliability of the SEN+2
0.99	0.932065	0.950334	0.897863
0.98	0.868126	0.901462	0.809831
0.96	0.751447	0.806756	0.667692
0.95	0.698337	0.761192	0.610132
0.94	0.648478	0.716965	0.559696
0.92	0.557847	0.632844	0.475794
0.90	0.478297	0.554872	0.408837

Table2 Broadcast reliability comparison



**Network reliability** is the ability of interconnecting all inputs to all outputs can be demonstrated by the network reliability. Network reliability of the 8x8 SEN, SEN+, and SEN+2 is evaluated in this section. Since SEN is a single-path MIN, the failure of any switch will cause the system failure. Therefore, from the reliability

point of view, there are  $(N/2)$   $(\log_2 N)$  SEs in series. The reliability of an NXN SEN is given by

$$R_n(\text{SEN}) = (r)^{(N/2)(\log_2 N)}.$$

As for 8X8 SEN system, the network reliability for  $N = 8$  can be calculated as

$$R_n(\text{SEN}) = (r)^{12}.$$

The network reliability of the 8X8 SEN+ (consists of four stages: stage 0, 1, 2, and 3) can be calculated as follows:

1. All SEs in stages 0 and 3 must be working for the system to be operational. For reliability evaluation, it can be assumed that these SEs are in series. Therefore, the reliability for stages 0 and 3 is equal to  $r^8$ .
2. In stage 1, consider the state of each switch as conditional and proceed with all possible combinations through stage 2 that make the input-output connections. A k out of n redundancy system is applied where there is at least k out of n components to function for the system to work.
3. Every SE has only two possible states, working or failed. Hence, the network reliability of the 8X8 SEN+ for  $N = 8$  can be derived as

$$R_n(\text{SEN+}) = (r^8) \text{ (the sum of all reliability in the stages 1 and 2),}$$

$$= 2r^{12} + 4r^{14} - 8r^{15} + 3r^{16}.$$

It is noted that the network reliability of the 8X8 SEN+ is much higher than that of the 8X8 SEN for all reliability values of the SEs. Similarly Network reliability can be computed for SEN+2

$$R_n(\text{SEN} + 2) = r^{20} + 4r^{19}(1 - r) + 36r^{18}(1 - r)^2$$

$$+ 120r^{17}(1 - r)^3 + 168r^{16}(1 - r)^4$$

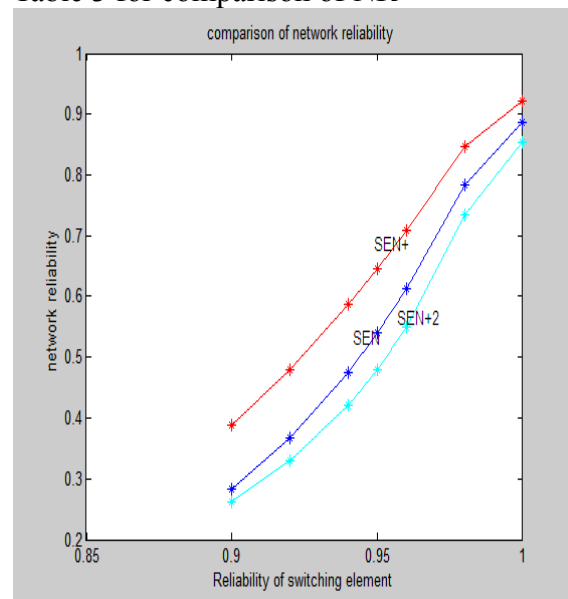
$$+ 2r^{16}(1 - r)^2 + 96r^{15}(1 - r)^5$$

$$+ 20r^{15}(1 - r)^3 + 16r^{14}(1 - r)^6$$

$$+ 14r^{14}(1 - r)^4 + 4r^{13}(1 - r)^3.$$

SE reliability	Network reliability of the SEN	Network reliability of the SEN+	Network reliability of the SEN+2
0.99	0.886385	0.921659	0.854252
0.98	0.784717	0.846842	0.733541
0.96	0.612710	0.708630	0.549891
0.95	0.540360	0.645470	0.480112
0.94	0.475920	0.586322	0.421465
0.92	0.367666	0.479906	0.329657
0.90	0.282430	0.388708	0.262133

Table 3 for comparison of NR



The basic parameter taken into consideration while evaluating TR, BR, NR is  $r$  which is defined as

$$R(t) = e^{-\lambda t}$$

### Failure Distribution Function

$$F(t) = 1 - e^{-\lambda t}$$

### Density Function of Failure

$$f(t) = \lambda e^{-\lambda t}$$

### Failure Rate

$$z(t) = \frac{f(t)}{R(t)} = \lambda$$

### Mean Time To Failure

$$MTTF = \int_0^{\infty} e^{-\lambda t} dt = \frac{1}{\lambda}$$

### System Reliability

$$\begin{aligned} R_{\text{system}}(t) &= \prod_{i=1}^n R_i(t) \\ &= e^{-\lambda_1 t} e^{-\lambda_2 t} e^{-\lambda_3 t} \dots e^{-\lambda_n t} \\ &= e^{-(\lambda_1 + \lambda_2 + \lambda_3 + \dots + \lambda_n)t} \\ &= e^{\left( -t \sum_{i=1}^n \lambda_i \right)} \end{aligned}$$

For 8X8 SEN+ (Network Reliability)

$$R_{\text{SEN+}}(t) = 2e^{-12 \int_0^t \lambda(\tau) d\tau} + 4e^{-14 \int_0^t \lambda(\tau) d\tau} - 8e^{-15 \int_0^t \lambda(\tau) d\tau} + 3e^{-16 \int_0^t \lambda(\tau) d\tau}$$

For exponential distribution function

$$MTTF_E = \sum_i \frac{a_i}{i\lambda_E}$$

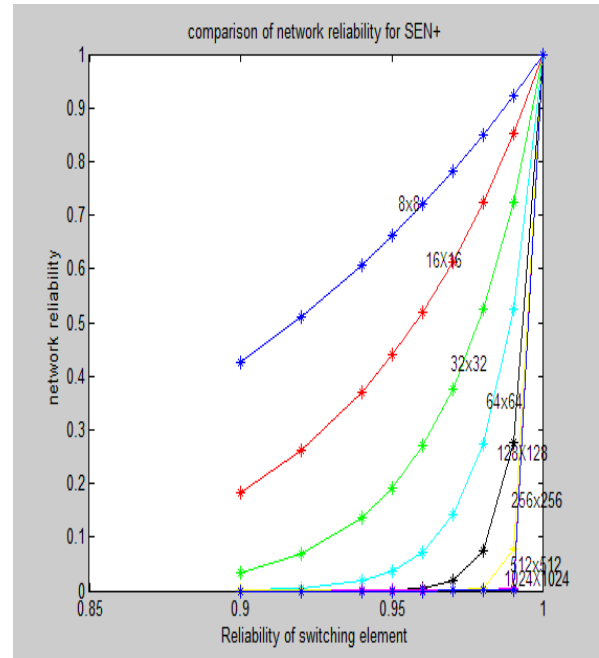
$$MTTF_E = \left( \frac{2}{12} + \frac{4}{14} - \frac{8}{15} + \frac{3}{16} \right) \frac{1}{\lambda_E} = \frac{179}{1680\lambda_E}$$

Similarly value of  $MTTF * \lambda$  can be evaluated for SEN, SEN+ (8X8) which are summarized in the table given below.

Network type	MTTF * $\lambda$		
	TR	BR	NR
SEN	1/3	1/7	1/8
SEN+	1/3	149/990	179/1680

Table 4.  $MTTF * \lambda$  for SEN, SEN+ (8X8)

Now effect of reliability on network size is taken into consideration and hence size of network is varied from 8X8 to 1024X1024 as shown below



### Conclusion :

Graph are plotted for SEN, SEN+, SEN+2 terminal reliability in response to table 1. From these graphs this can be concluded that by adding a single stage reliability thus improves but if we add another stage ie SEN+2 reliability reduces thus there is not a direct relation between additional paths and the terminal reliability increment in SEN

because the additional paths may increase the links complexity of the network, leading to a higher failure. Hence, it can be concluded that adding one additional stage to the SEN is more efficient way to improve terminal reliability than an addition of two stages to the SEN. Graphs are plotted for SEN, SEN+, SEN+2 broadcast reliability in response to table 2. Broadcast reliability is highest for SEN+ and lowest for SEN+2. Graphs are also plotted for SEN, SEN+, SEN+2 network reliability in response to table 3. Network reliability is highest for SEN+ and lowest for SEN+2. If we reconsider the table 1, 2 and 3 we can conclude reliability of Shuffle exchange network topology (terminal, broadcast, network) is improved by adding one stage and is lowered with further addition of intermediate stage.

Further considering the reliability tables an important conclusion can be derived if the switching reliability value lies between  $0.99 - 1$  and desired  $MTTF * \lambda$  can be computed hence better performance can be attained. Thus replacing the existing switching device i.e. CMOS transistor with switching device having better switching reliability, reliability can be improved. This scheme can be solution in case of supercomputing in which if the information to be transferred between processors and memory is transferred in optical domain and switching device between processor and memories are passive device like MEOMS, directional coupler further improvement can be attained in processing speed of supercomputer if WDM technique is being followed. Thus the desired value of  $MTTF * \lambda$  of switching device should be comparable with the value so computed in the

table 4.

## References

1. "Reliability analysis of shuffle exchange network systems", Indra Gunwana :Reliability engineering & system safety 13, October 2004
2. "Permutation Capability and connectivity of Enhanced Multistage Interconnection Network (E-MIN), Subramanyamb Aridge, 2006 IEEE
3. "Large-Capacity Strictly Nonblocking Optical Cross-Connects Based on Microelectrooptomechanical Systems (MEOMS)" Switch Matrices: Reliability Performance Analysis ", Lena Wosinska, Member, JOURNAL OF LIGHTWAVE TECHNOLOGY, VOL. 19, NO. 8, AUGUST 2006
4. "Reconfigurable optical Interconnection System Supporting Concurrent Application-Specific Parallel Computing ", Sacki Agelis and Magnus Jonsson :Proceedings on the international symposium on computer architecture and high performance computing (SBAC-PAD'06) 2006
5. "Fault-tolerant DSM on the SOME-Bus Multiprocessor Architecture", Constantine Katsinis and Diana Hecht :IEEE IPDPS, Santa Fe, New Mexico, April 2006
6. "A modified algorithm for computing the upper-bound reliability of computer network", Y.B Mandy et al. Applied mathematics and computation 2005